

## Simultaneous Equations

Patrons are reminded to try to use the substitution method where possible. Look for an  $x$  or  $y$  to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{aligned} 3x + y &= 7 \\ 2x + 5y &= 2 \end{aligned}$$

From the first we can see that  $y = 7 - 3x$ . Substituting this into the second we find  $2x + 5(7 - 3x) = 2$  which solves to  $x = \frac{33}{13}$ . We then place this value into  $y = 7 - 3x$  to discover  $y = -\frac{8}{13}$ . So  $(x, y) = (\frac{33}{13}, -\frac{8}{13})$

Present answers in the form  $(x, y) = (-1, \frac{1}{2})$ . (Don't forget the brackets!)

$$1. \begin{aligned} x + y &= 3 \\ x - y &= 2 \end{aligned} \quad \boxed{(x, y) = (\frac{5}{2}, \frac{1}{2})} \quad 13. \begin{aligned} 5x + 4y &= 1 \\ x - 3y &= 0 \end{aligned} \quad \boxed{(x, y) = (\frac{3}{19}, \frac{1}{19})}$$

$$2. \begin{aligned} x - y &= 4 \\ x + y &= 7 \end{aligned} \quad \boxed{(x, y) = (\frac{11}{2}, \frac{3}{2})} \quad 14. \begin{aligned} x + y &= -7 \\ 2x + 3y &= 4 \end{aligned} \quad \boxed{(x, y) = (-25, 18)}$$

$$3. \begin{aligned} 2x + y &= 3 \\ x - 2y &= 2 \end{aligned} \quad \boxed{(x, y) = (\frac{8}{5}, -\frac{1}{5})} \quad 15. \begin{aligned} 2x + y &= 2 \\ x - 6y &= 1 \end{aligned} \quad \boxed{(x, y) = (1, 0)}$$

$$4. \begin{aligned} x + 2y &= 4 \\ 3x - 2y &= -7 \end{aligned} \quad \boxed{(x, y) = (-\frac{3}{4}, \frac{19}{8})} \quad 16. \begin{aligned} 3x + 2y &= 1 \\ y - 2x &= -3 \end{aligned} \quad \boxed{(x, y) = (1, -1)}$$

$$5. \begin{aligned} 5x - y &= 4 \\ 4x - 5y &= 0 \end{aligned} \quad \boxed{(x, y) = (\frac{20}{21}, \frac{16}{21})} \quad 17. \begin{aligned} a - b &= 2 \\ 3a - 2b &= -4 \end{aligned} \quad \boxed{(a, b) = (-8, -10)}$$

$$6. \begin{aligned} 2x - 3y &= 5 \\ 3x + 2y &= 2 \end{aligned} \quad \boxed{(x, y) = (\frac{16}{13}, -\frac{11}{13})} \quad 18. \begin{aligned} 4x + 7y &= 10 \\ 3x - y &= -2 \end{aligned} \quad \boxed{(x, y) = (-\frac{4}{25}, \frac{38}{25})}$$

$$7. \begin{aligned} 4x - 2y &= -9 \\ 3x + 5y &= 3 \end{aligned} \quad \boxed{(x, y) = (-\frac{3}{2}, \frac{3}{2})} \quad 19. \begin{aligned} 2x + 3y &= 1 \\ 3x - 4y &= 2 \end{aligned} \quad \boxed{(x, y) = (\frac{10}{17}, -\frac{1}{17})}$$

$$8. \begin{aligned} x - 2y &= 2 \\ 3x + 2y &= 1 \end{aligned} \quad \boxed{(x, y) = (\frac{3}{4}, -\frac{5}{8})} \quad 20. \begin{aligned} 5x + y &= 7 \\ 4x - \frac{1}{2}y &= 2 \end{aligned} \quad \boxed{(x, y) = (\frac{11}{13}, \frac{36}{13})}$$

$$9. \begin{aligned} 3x - y &= 3 \\ 2x + 3y &= -1 \end{aligned} \quad \boxed{(x, y) = (\frac{8}{11}, -\frac{9}{11})} \quad 21. \begin{aligned} x + 5y &= 0 \\ 3x + 4y &= -1 \end{aligned} \quad \boxed{(x, y) = (-\frac{5}{11}, \frac{1}{11})}$$

$$10. \begin{aligned} y - 2x &= 5 \\ 5x - 7y &= 2 \end{aligned} \quad \boxed{(x, y) = (-\frac{37}{9}, -\frac{29}{9})} \quad 22. \frac{x + y}{2} - \frac{x - y}{3} = 1 \quad \boxed{(x, y) = (\frac{22}{7}, \frac{4}{7})}$$

$$11. \begin{aligned} x + 3y &= 4 \\ 5x - 2y &= 6 \end{aligned} \quad \boxed{(x, y) = (\frac{26}{17}, \frac{14}{17})} \quad 23. \frac{x + 2y}{3} - \frac{x - 3y}{7} = x \quad \boxed{(x, y) = (\frac{23}{37}, \frac{17}{37})}$$

$$12. \begin{aligned} 4x - y &= 2 \\ 3x + 4y &= 1 \end{aligned} \quad \boxed{(x, y) = (\frac{9}{19}, -\frac{2}{19})} \quad 24. \begin{aligned} x + ay &= 0 \\ 2x + 3y &= -1 \end{aligned} \quad \boxed{(x, y) = (\frac{a}{3-2a}, \frac{1}{2a-3})}$$

$$25. \begin{cases} kx + y = 4 \\ 2x - 3y = 2 \end{cases}$$

$$(x, y) = \left( \frac{14}{2+3k}, \frac{8-2k}{2+3k} \right)$$

$$30. \begin{cases} 4x + y = 3 \\ ax + by = c \end{cases}$$

$$(x, y) = \left( \frac{c-3b}{a-4b}, \frac{3a-4c}{a-4b} \right)$$

$$26. \begin{cases} ax + 4y = 6 \\ bx - y = 5 \end{cases}$$

$$(x, y) = \left( \frac{26}{a+4b}, \frac{6b-5a}{a+4b} \right)$$

$$31. \begin{cases} x + by = 2 \\ \frac{x + ay}{2} - \frac{x - y}{3} = 1 \end{cases}$$

$$(x, y) = \left( \frac{6a+4-6b}{3a+2-b}, \frac{4}{3a+2-b} \right)$$

$$27. \begin{cases} kx + y = 1 \\ 5x - ky = m \end{cases}$$

$$(x, y) = \left( \frac{m+k}{5+k^2}, \frac{5-km}{5+k^2} \right)$$

$$28. \begin{cases} x + y = 1 \\ ax + by = 1 \end{cases}$$

$$(x, y) = \left( \frac{1-b}{a-b}, \frac{a-1}{a-b} \right)$$

$$32. \begin{cases} \frac{ax+1}{2} + \frac{by+2}{3} = 1 \\ \frac{5x+1}{3} + \frac{ay+1}{2} = 1 \end{cases}$$

$$(x, y) = \left( \frac{3a+2b}{20b-9a^2}, \frac{3a+10}{9a^2-20b} \right)$$

$$29. \begin{cases} x + ay = 3 \\ ax + by = 4 \end{cases}$$

$$(x, y) = \left( \frac{3b-4a}{b-a^2}, \frac{4-3a}{b-a^2} \right)$$

$$33. \begin{cases} 23x + 21y = 1 \\ 21x + 23y = -1 \end{cases}$$

$$(x, y) = \left( \frac{1}{2}, -\frac{1}{2} \right)$$

Now solve the following simultaneous equations in three unknowns.

1.

$$\begin{aligned}x + y + z &= 1 \\2x + 3y + z &= 6 \\x - y + 2z &= -5\end{aligned}$$

$$x = 1, y = 2, z = -2$$

2.

$$\begin{aligned}x + y + z &= 1 \\x - y + 2z &= 2 \\2x + 3y + 3z &= 3\end{aligned}$$

$$x = 0, y = 0, z = 1$$

3.

$$\begin{aligned}2a + b + 3c &= -7 \\a - b + 2c &= -4 \\3a + 2b - c &= 11\end{aligned}$$

$$a = 3, b = -1, c = -4$$

4.

$$\begin{aligned}2x + 3y - z &= 2 \\4x - y + 2z &= 5 \\2x + y - 3z &= -4\end{aligned}$$

$$x = \frac{1}{2}, y = 1, z = 2$$

5.

$$\begin{aligned}x - y + z &= 1 \\2x + 2y + 3z &= 1 \\x - y - 4z &= 2\end{aligned}$$

$$x = 1, y = -\frac{1}{5}, z = -\frac{1}{5}$$

6.

$$\begin{aligned}a + 2b + c &= 3 \\2a + b + c &= 1 \\a - b + 2c &= 0\end{aligned}$$

$$a = -\frac{2}{3}, b = \frac{4}{3}, c = 1$$

7.

$$\begin{aligned}p + q + r &= -1 \\2p + q + 2r &= -1 \\p + 3q &= 1\end{aligned}$$

$$p = 4, q = -1, r = -4$$

8.

$$\begin{aligned}x + y - z &= 1 \\x - 2y + 3z &= 0 \\x - y + 2z &= -1\end{aligned}$$

$$x = 2, y = -5, z = -4$$

9.

$$\begin{aligned}4x - 5y + 2z &= -2 \\5x + 7y + 3z &= 3 \\2x + 3y + z &= 1\end{aligned}$$

$$x = -\frac{8}{11}, y = \frac{4}{11}, z = \frac{15}{11}$$